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PRIMARY PAPER

A comparison of the accuracy of an adjusted fuzzy time series forecasting method with the traditional method application to Thailand rubber price

Kanittha Yimnak *

Department of Mathematics and Statistics, Dhurakij Pundit University, Thailand

Index Terms
Fuzzy Time Series
Thailand Rubber Price
Symmetric Trapezoidal Fuzzy Numbers
Traditional Method
Uncertain Data.

Abstract—The modified approach on fuzzy time series, which is represented by [3], is applied to rubber price in Thailand. The developed forecasting method corresponds to the uncertain data. The nearest symmetric trapezoidal fuzzy numbers are used to further enhance the forecasting accuracy. The accuracy of this method is compared to the traditional method and actual values by the Mean Absolute Percentage Error (MAPE). The results show that the forecasts by the developed fuzzy time series forecasting method is more accuracy than the traditional method.

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I. INTRODUCTION

The accuracy of forecasting techniques isessential, especially economy and social data, because it leads toset the policy of developing country. The mostly time series data are uncertain data. The accuracy of interpretation of uncertain data is interest. Normally, The traditional time series forecasting methods such as moving average, smoothing exponential, Box-Jenkins and the other classical methods are well-known for time series. However, these methodsdo not satisfy for forecasting problem in which the time series data are linguistic values. In addition, for uncertain time series data, there will be less accurate when using these methods. Fuzzy time series forecasting method is a good alternative for uncertain data. It has been widely such as forecasting the weather, earthquakes, stock fluctuations and any phenomenon indexed by variables that change unpredictably in time. The nearest trapezoidal approximation is proposed by [1]. This approximation operator preserving expected interval and possesses many desired properties. In some situations their operator many fail to lead a trapezoidal fuzzy number. In 2007 [2], suggested the efficient process of trapezoidal approximation operator. [3] found that the nearest symmetric trapezoidal fuzzy numbers enhanced the forecasting accuracy for fuzzy time series forecasting method. The accuracy of forecasting time series data in this process increases. In this research, the price of raw rubber of Thailand is used for numerical experiment. The accuracy of the presented method and the traditional method are compared by the Mean Absolute Percentage Error (MAPE).

* Corresponding author: Kanittha Yimnak
E-mail: kanittha.yim@dpu.ac.th

II. TRADITIONAL METHODS

There are many traditional forecasting methods,i.e., the simple linear regression model satisfying the time series that components of trend, Box-Jenkins method satisfying a stationary time series that feature finite order Moving Average (MA(q)), finite order Autoregressive (AR(p)) model, mixture of finite order Autoregressive And Moving Average ARMA(p,q) and ARIMA(p,d,q) model for nonstationary data using ARMA models defined on the dth difference of the original process and the time series data comprising influenced by the seasonality-periodic or SARIMA(p,d,q)(P,D,Q)$_s$ model that repeat with about the same intensity each year.

The AR(p) model is the p^{th} order autoregressive time series and X_k can be solved by the following equation[4].

$$\sum_{i=0}^{p} \rho_i X_{k-i} = \varepsilon_k, \qquad k = 0, \pm 1, \pm 2, \ldots, \tag{1}$$

where $\rho_0 \neq 0, \rho_p \neq 0$ and the$\varepsilon_k$are typically assumed to be uncorrelated $(0, \sigma^2)$ random variables (i.e.$E[\varepsilon_0] = 0, E[\varepsilon_0^2] = \sigma^2$). Thus, the AR(1) process is a first order autoregressive time series and most commonly defined by the following equations

$$X_k = \rho X_{k-1} + \varepsilon_k, \quad k = 0, \pm 1, \pm 2, \ldots \tag{2}$$

which is simply a first order linear difference equation. Note that X_k is represented as alinear function of X_{k-1} and ε_kIt can also be shown that for this model [5].

1. The Theoretical Autocorrelation Function (TACF) dies down (that is, the values $r_1, r_2, \ldots$ decrease in a steady fashion). For the working series $X_b, X_{b+1}, \ldots, X_n$: the simple autocorrelation at lag k is

$$r_k = \frac{\sum_{t=b}^{n-k}(X_t - \overline{X})(X_{t+k} - \overline{X})}{\sum_{t=b}^{n}(X_t - \overline{X})^2}, \tag{3}$$

where

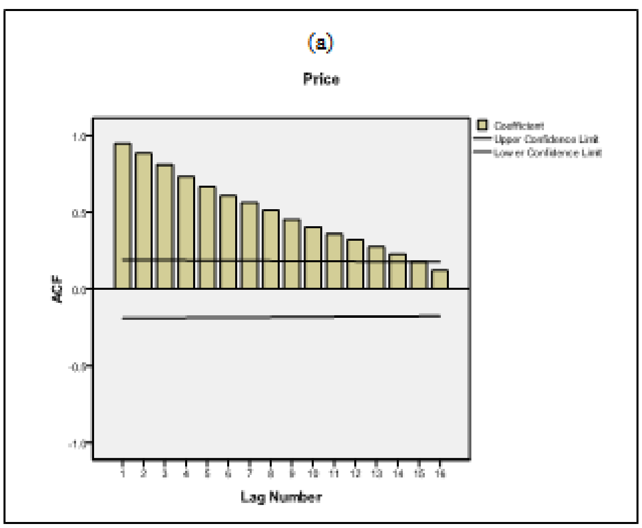
$$\overline{X} = \frac{\sum_{t=b}^{n} X_t}{n - b + 1} \tag{4}$$

2. The Theoretical Partial Autocorrelation Function (TPACF) has a nonzero partial autocorrelations at lag *1* and has zero partial autocorrelations at all lags after lag1 (that is, cuts off after lag 1). Said equivalently, $r_{kk} \neq 0$ for $k = 1$, $r_{kk} = 0$for$k > 1$,

$$r_{kk} = \begin{cases} r_1 & ; k = 1 \\ \dfrac{r_k - \sum_{j=1}^{k-1} r_{k-1,j} r_{k-j}}{1 - \sum_{j=1}^{k-1} r_{k-1,j} r_j} & ; k = 2, 3, \ldots \end{cases}, \tag{5}$$

where$r_{kj} = r_{k-1,j} - r_{kk} r_{k-1,k-j}$ for $j = 1, 2, \ldots, k-1$

The TACF and TPACF of the numerical experiment show in figure 1.



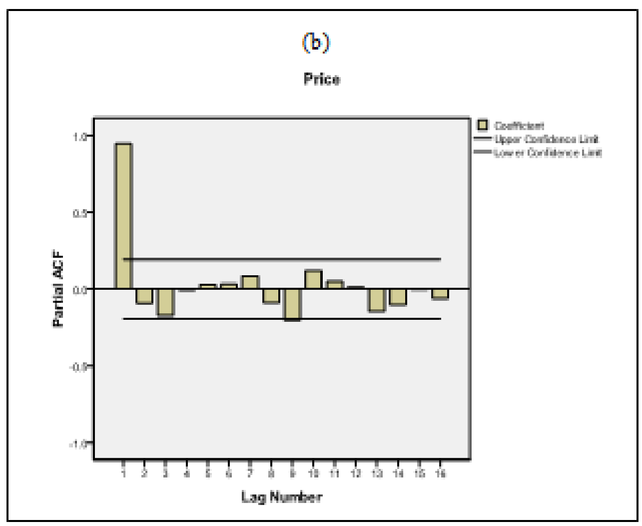


Fig. 1. The TACF and TPACF of Thailand rubber price; (a) TACF: (b) TPACF

In this study, the AR(1) model corresponding to Thailand rubber price data is chosen for comparison to an adjusted fuzzy time series forecasting method. For the AR(1) model, the $\varepsilon_k's$ are considered as a random

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variables. Moreover, they areoften mentioned in the error terms or innovations. For the difference equation (2), $X_k$can be solved byknowing the value of X_{k-1}, and X_{k-1} is gained from knowing X_{k-2} and so on,X_0 is an initial value that is k-time periods prior.

$$\begin{aligned} X_k &= \rho X_{k-1} + \varepsilon_k \\ &= \rho(\rho X_{k-2} + \varepsilon_{k-1}) + \varepsilon_k \\ &= \rho^2 X_{k-2} + \rho\varepsilon_{k-1} + \varepsilon_k \\ &= \rho^3 X_{k-3} + \rho^2\varepsilon_{k-2} + \rho\varepsilon_{k-1} + \varepsilon_k \\ &\vdots \\ &= \rho^N X_{k-N} + \rho^{N-1}\varepsilon_{k-(N-1)} + \ldots + \rho\varepsilon_{k-1} + \varepsilon_k \end{aligned}$$

Thus, for every N

$$X_k = \rho^N X_{k-N} + \sum_{i=0}^{N-1} \rho^i \varepsilon_{k-i,} \qquad (6)$$

The limit as $N \to \infty$ of equation (6) and for $|\rho| < 1$ indicates $X_k = \sum_{i=0}^{\infty} \rho^i \varepsilon_{k-i}$must be the solution of the difference equation, assuming that the infinite sum $\sum_{i=0}^{\infty} \rho^i \varepsilon_{k-i}$ exists. If ε_k is stationary, then $\sum_{i=0}^{\infty} \rho^i \varepsilon_{k-i}$ is also stationary. The distribution of our error terms are found before finding the distribution of our data. To find the distribution of our data, we need to first find the distribution of our error terms. Therefore, we solve for $\epsilon_{k.}$ Recall,

$$X_k = \rho X_{k-1} + \epsilon_{k.} \qquad (7)$$

$$\hat{\epsilon}_k = X_k - \hat{\rho} X_{k-1} \qquad (8)$$

We obtain our least squares estimator for ρ:

$$\hat{\rho} = \frac{\sum_{k=2}^{n} X_k X_{k-1}}{\sum_{k=2}^{n} X_{k-1}^2}. \qquad (9)$$

By estimating $\hat{\rho}$, we can see the correlation between each observation from time period to time period.

A. The Fuzzy Time Series Forecasting Method Based on the Nearest Symmetric Trapezoidal Fuzzy Numbers

[3] proposed 8 steps for this modified method.

Step 1: Assemble the fuzzy time series data Av_t.

Step 2: Determine D_{max} and D_{min} are the maximum and the minimum among all Av_t, respectively. Theuniverse of discourse two small numbers D_1 and D_2 are assigned as

$$U = [D_{min} - D_1, D_{max} + D_2]$$

Step 3: The universe is divided in to seven equal length intervals $U_i, i = 1,2,..,7$ [6] according to the distribution of the historical data.$U_i$is divided into intervals of different length and denotesv_j.

$$v_1 = [d_1, d_2], \quad v_2 = [d_2, d_3], \ldots, v_m = [d_m, d_{m+1}]$$

Step 4: Define trapezoidal fuzzy number. The fuzzy sets which are defined on trapezoidal fuzzy numbers are as follows:

$$A_1 = [d_0, d_1, d_2, d_3]$$

$$A_2 = [d_1, d_2, d_3, d_4]$$

$$\vdots$$

$$A_{m-1} = [d_{m-2}, d_{m-1}, d_m, d_{m+1}]$$

$$A_m = [d_{m-1}, d_m, d_{m+1}, d_{m+2}]$$

Step 5: Classify all the data in to the corresponding fuzzy numbers. The time series data belongs to the fuzzy number A_j when the value of time series data is located in the range of v_j.

Step 6: Defined the fuzzy logical relationship under the condition that if $Av_t(t-1) = A_i$ and $Av_t(t) = A_j$ then a fuzzy logical relationship can be defined as $A_i \to A_j$, where A_i and A_j are called the left hand side and the right hand side of the fuzzy logical relationship respectively [7,8,9].

Step 7: Arrange the fuzzy logical relationships in to the fuzzy logical relationship groups

based on the same fuzzy number on the left hand sides of the fuzzy logical relationships. If the transition happens to the same fuzzy set, make a separate logical relationship group. The fuzzy logical relationship groups are like the following;

$$A_j \to A_{k1}, A_j \to A_{k2}, \ldots, A_j \to A_{kp}$$

Step 8: Forecast time series data. The forecasted value at time t, Fv_t, can be solved by the following three heuristic rules.Suppose the fuzzy number Av_t at time$t-1$isA_j.

Note: For the trapezoidal fuzzy number $A = (t_1, t_2, t_3, t_4)$ where $[t_2, t_3]$ is core of A, $t_1$is left width and t_4 is right width, then the nearest symmetric trapezoidal fuzzy number for A is:

$$\left(t_2 + \frac{t_4 - t_1}{4}, t_3 + \frac{t_4 - t_1}{4}, \frac{t_1 + t_4}{2}\right) [7,8,9]$$

Rule 1: If the fuzzy logical relationship group of A_j is empty, $A_j \to \varphi$ or $A_j \to A_j$, then the forecasted valueFv_t is $R[NSTFN(A_j)]$

Note: A ranking function is a map from the set of fuzzy numbers $F(R)$ into areal line and is defined by

$$R(A) = \frac{a+b+c+d}{4}, \qquad A = (a, b, c, d)$$

Rule 2: If the fuzzy logical relationship group of A_j is one to one, i.e., $A_j \rightarrow A_k$ then the forecasted value $Fv_t$is $R[NSTFN(A_k)]$.
Rule 3: If the fuzzy logical relationship group of A_j is one to many, i.e.,
$A_j \rightarrow A_{k1}, A_j \rightarrow A_{k2}, \ldots, A_j \rightarrow A_{kp}$ then the value of Fv_t is calculated as:

$$Fv_t = R\left[\frac{NSTFN(A_{k1}) + NSTFN(A_{k2}) + \ldots + NSTFN(A_{kp})}{p}\right]$$

B. Numerical Example

This section presents the steps for forecasting the prices of raw rubber in Thailand. The Mean Absolute Percentage Error (MAPE) is used to evaluate the forecasting. The formula is:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Av_t - Fv_t}{Av_t}\right| \times 100$$

Step 1: The rubber prices are collected from January2007 to December 2015 [10] as shown in table 2.
Step 2: The maximum and minimum data of the rubber price are 172.62 and 34.51 respectively. Let $D_1 = 1.51, D_2 = 0.38$. Then$U = [33,173]$
Step 3: Divide U into seven interval $U_i, i = 1,2,..,7$ which equal length of 140 i.e.
$U_1 = [33,53], U_2 = [53,73], \ldots, U_7 = [153,173]$

TABLE 1
THE FREQUENCY DISTRIBUTION OF U_1 TO U_7ARE AS THE FOLLOWING

Interval	U_1	U_2	U_3	U_4	U_5	U_6	U_7
No of historical the rubber price	24	26	30	17	5	4	2

Divide the intervals $U_i, i = 1,2,..,7$ as follows:
$v_1 = [33,33.8), \ldots, v_{24} = [52.1,53)$ with length 0.8 and 0.9 alternatively.
$v_{25} = [53,53.7), \ldots, v_{50} = [72.2,73)$ with length 0.7 and 0.8 alternatively.
$v_{51} = [73,73.6), \ldots, v_{80} = [92.3,93)$with length 0.6 and 0.7 alternatively.
$v_{81} = [93,94.1), \ldots, v_{97} = [111.8,113)$ with length 1.1 and 1.2 alternatively.
$v_{98} = [113,117), \ldots, v_{102} = [129,133)$ with length of 4.
$v_{103} = [133,138), \ldots, v_{106} = [148,153)$with length of 5.
$v_{107} = [153,163), v_{108} = [163,173)$ with length of 10.
Step4: The fuzzy trapezoidal numbers can be then defined by
$A_1 = [32.2 \quad 33 \quad 33.8 \quad 34.6\,]$
$A_2 = [33 \quad 33.8 \quad 34.6 \quad 35.4\,]$
⋮
$A_{107} = [148 \quad 153 \quad 163 \quad 173\,]$
$A_{108} = [153 \quad 163 \quad 173 \quad 183\,]$

Step5: Fuzzify the rubber prices as shown in table2.
Step6: According to the fuzzified the rubber prices, the fuzzy logical relationships are derived as follows:

TABLE 2
FUZZY LOGICAL RELATIONSHIP

$A_{41} \rightarrow A_{51}$	$A_{51} \rightarrow A_{47}$	$A_{47} \rightarrow A_{51}$	$A_{51} \rightarrow A_{53}$	$A_{53} \rightarrow A_{47}$	$A_{47} \rightarrow A_{37}$
$A_{37} \rightarrow A_{40}$	$A_{40} \rightarrow A_{42}$	$A_{42} \rightarrow A_{49}$	$A_{49} \rightarrow A_{56}$	…	$A_5 \rightarrow A_5$

Step 7: Create the fuzzy logical relationship groups as follows:
$A_{41} \rightarrow A_{35}, A_{46}, A_{51}$
$A_{51} \rightarrow A_{47}, A_{53}$
$A_{47} \rightarrow A_{37}, A_{49}, A_{51}, A_{55}$
⋮
$A_{15} \rightarrow A_{15}$
$A_{98} \rightarrow A_{102}$
Step8: Calculate the forecasted the rubber prices. For example
$[Dec - 10]$: The fuzzified enrollment of $[Nov. - 10]$ isA_{98} and the corresponding fuzzy logical relationship group $A_{98} \rightarrow A_{102}$
$A_{102} = [125 \;\; 129 \;\; 133 \;\; 138]$.By note $t_2 = 129$; $t_3 = 133$; $t_1 = 129 - 125 = 4$; $t_4 = 138 - 133 = 5$
$\frac{t_4 - t_1}{4} = 0.25, \quad \frac{t_4 + t_1}{2} = 4.5$
$NSTFN(A_{102})$
$= [129.25 - 4.5 \quad 129 + 0.25 \quad 133 + 0.25 \quad 142.25 - 4.5]$
$= [124.75 \quad 129.25 \quad 133.25 \quad 137.75]$
According to rule3, theFv_t for $[Dec - 10]$ is
$Fv_t = R[NSTFN(A_{102})] = 131.250$
$[Sep - 08]$and$[Jun - 08]$: the fuzzified rubber prices of $[Sep - 08]$and $[Jun - 08]$is A_{77}.The fuzzy logical relationship group is $A_{77} \rightarrow A_{85}, A_{76}$.
$A_{85} = [96.3 \quad 97.4 \quad 98.6 \quad 99.8]$. $NSTFN(A_{85})$

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$= [96.275 \quad 97.425 \quad 98.625 \quad 99.775]$

$A_{76} = [88.8 \quad 89.5 \quad 90.2 \quad 90.9], \mathrm{NSTFN}(A_{76})$
$= [88.8 \quad 89.5 \quad 90.2 \quad 90.9]$

According to rule3, the Fv_t for [Sep – 08] and [Jun – 08] is

$$Fv_t = R\left[\frac{96.275+88.8}{2}, \frac{97.425+89.5}{2}, \frac{98.625+90.2}{2}, \frac{99.775+90.9}{2}\right] = 93.938$$

For figure 2, the actual data , AR(1) forecasting model and proposed method are coincide. However, the MAPE using proposed method is less than the AR(1) method (see in Table1.). In addition, the MAPE using tradition method is about 2.10 times which compares to the MAPE using proposed method.

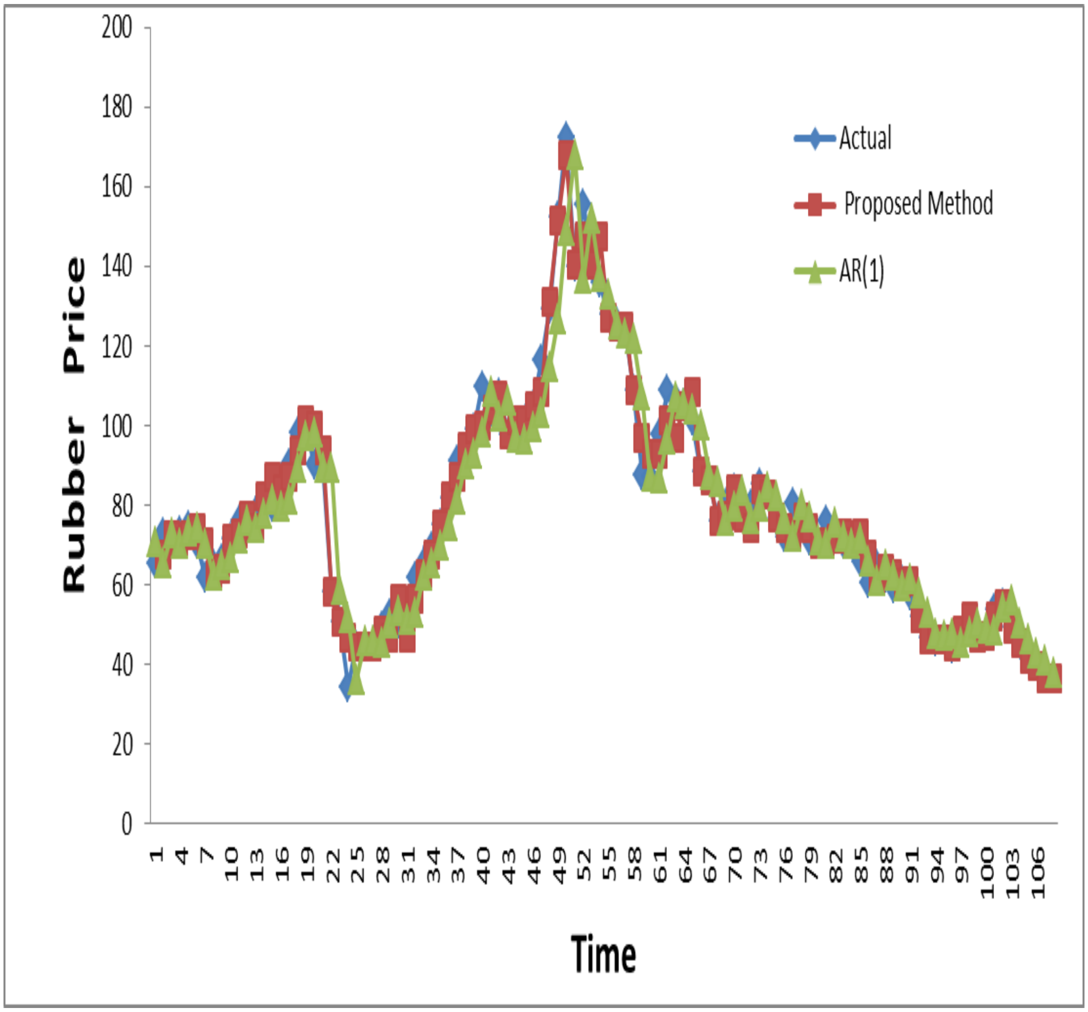

Fig. 2. The actual data the forecasts by AR (1) and the proposed methods.

TABLE 3
A COMPARISON BETWEEN THE MAPE BY THE PROPOSED METHOD AND THE AR(*1*) METHOD

Index	AR (*1*)	Proposed method	Ratio of AR(1) to proposed method
MAPE	7.551	3.597	2.10 : 1

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TABLE 4
FUZZIFIED AND FORECASTED THE THAILAND RUBBER PRICES

Time	Rubber Price	Fuzzified Rubber Price	Fv_t	MAPE	Time	Rubber Price	Fuzzified Rubber Price	Fv_t	MAPE
Jan-07	65.43	A41			Jul-11	127.9	A101	127.000	0.0070
Feb-07	73.1	A51	67.750	0.0732	Aug-11	125.74	A101	125.000	0.0059
Mar-07	70.19	A47	72.350	0.0308	Sep-11	123.96	A100	125.000	0.0084
Apr-07	73.35	A51	70.738	0.0356	Oct-11	109.16	A94	108.800	0.0033
May-07	74.67	A53	72.350	0.0311	Nov-11	87.88	A73	97.075	0.1046
Jun-07	70.56	A47	74.150	0.0509	Dec-11	87.43	A73	92.888	0.0624
Jul-07	61.87	A37	70.738	0.1433	Jan-12	97.81	A85	92.888	0.0503
Aug-07	64.9	A40	63.000	0.0293	Feb-12	108.9	A94	101.606	0.0670
Sep-07	66.46	A42	64.200	0.0340	Mar-12	106.95	A92	97.075	0.0923
Oct-07	71.72	A49	71.800	0.0011	Apr-12	105.54	A91	105.200	0.0032
Nov-07	76.22	A56	73.163	0.0401	May-12	101.48	A88	108.650	0.0707
Dec-07	74.58	A53	77.275	0.0361	Jun-12	88.44	A74	88.450	0.0001
Jan-08	78.35	A59	74.150	0.0536	Jul-12	86.43	A71	86.350	0.0009
Feb-08	81.93	A65	82.150	0.0027	Aug-12	76.17	A56	76.300	0.0017
Mar-08	79.84	A62	87.283	0.0932	Sep-12	79.9	A62	77.275	0.0329
Apr-08	81.88	A65	84.017	0.0261	Oct-12	84.15	A68	84.017	0.0016
May-08	90.31	A77	87.283	0.0335	Nov-12	76.84	A57	76.900	0.0008
Jun-08	98.45	A85	93.938	0.0458	Dec-12	80.29	A62	74.283	0.0748
Jul-08	99.43	A86	101.606	0.0219	Jan-13	85.37	A70	84.017	0.0159
Aug-08	90.37	A77	100.275	0.1096	Feb-13	82.8	A66	82.850	0.0006
Sep-08	90.09	A76	93.938	0.0427	Mar-13	76.7	A57	76.900	0.0026
Oct-08	58.13	A32	58.250	0.0021	Apr-13	72.03	A49	74.283	0.0313
Nov-08	50.97	A22	50.800	0.0033	May-13	80.52	A63	73.163	0.0914
Dec-08	34.51	A2	46.850	0.3576	Jun-13	77.16	A57	76.900	0.0034
Jan-09	44.69	A15	44.600	0.0020	Jul-13	71.05	A48	74.283	0.0455
Feb-09	44.93	A15	44.600	0.0073	Aug-13	70.53	A47	70.200	0.0047
Mar-09	44.49	A15	44.600	0.0025	Sep-13	75.92	A55	70.738	0.0683
Apr-09	49.56	A21	48.550	0.0204	Oct-13	72.16	A49	71.800	0.0050
May-09	52.74	A24	46.850	0.1117	Nov-13	70.38	A47	73.163	0.0395
Jun-09	50.55	A22	56.500	0.1177	Dec-13	71.95	A49	70.738	0.0169
Jul-09	52.27	A24	46.850	0.1037	Jan-14	65.75	A41	73.163	0.1127
Aug-09	61.88	A37	56.500	0.0869	Feb-14	60.69	A35	67.750	0.1163
Sep-09	65.29	A41	63.000	0.0351	Mar-14	64.81	A40	61.075	0.0576
Oct-09	69.66	A46	67.750	0.0274	Apr-14	61.9	A37	64.200	0.0372
Nov-09	75.15	A54	75.100	0.0007	May-14	59.19	A33	63.000	0.0644
Dec-09	81.97	A65	82.150	0.0022	Jun-14	60.94	A35	60.600	0.0056
Jan-10	91.29	A78	87.283	0.0439	Jul-14	57.36	A31	61.075	0.0648
Feb-10	94.15	A82	94.650	0.0053	Aug-14	52.01	A23	51.700	0.0060
Mar-10	99.44	A86	99.200	0.0024	Sep-14	46.67	A17	46.300	0.0079
Apr-10	110.01	A95	100.275	0.0885	Oct-14	46.05	A17	46.300	0.0054
May-10	103.77	A90	104.000	0.0022	Nov-14	46.66	A17	46.300	0.0077
Jun-10	107.91	A93	107.600	0.0029	Dec-14	44.3	A15	44.600	0.0068
Jul-10	98.04	A85	98.025	0.0002	Jan-15	46.94	A18	48.550	0.0343
Aug-10	97.63	A85	101.606	0.0407	Feb-15	49.64	A21	51.975	0.0470
Sep-10	100.85	A87	101.606	0.0075	Mar-15	48.28	A19	46.850	0.0296
Oct-10	104.81	A91	105.200	0.0037	Apr-15	47.42	A18	47.200	0.0046
Nov-10	116.58	A98	108.650	0.0680	May-15	53.79	A26	51.975	0.0337
Dec-10	129.42	A102	131.250	0.0141	Jun-15	55.3	A28	55.450	0.0027
Jan-11	152.59	A106	151.750	0.0055	Jul-15	49.24	A20	49.000	0.0049
Feb-11	172.62	A108	168.000	0.0268	Aug-15	45.45	A16	45.425	0.0006
Mar-11	140.12	A104	140.500	0.0027	Sep-15	41.59	A11	41.400	0.0046
Apr-11	155.79	A107	147.500	0.0532	Oct-15	39.64	A9	39.800	0.0040
May-11	140.39	A104	140.500	0.0008	Nov-15	36.49	A5	36.600	0.0030
Jun-11	135.89	A103	147.500	0.0854	Dec-15	36.8	A5	36.600	0.0054

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III. CONCLUSION AND DISCUSSION

The proposed method is suitable for Thailand rubber pricedata in sense of its accuracy. However, the traditional method is convenient method by using conventional package programing such as SPSS, SAS and MINITAB.For the time series data used for high precision forecasting, the proposed method is shown to be a good alternative because it produces high accuracy forecasting corresponding to the actual value.

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— This article does not have any appendix. —