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PRIMARY RESEARCH

Event-based networked control scheme for two-wheeled systems

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Abstract

This paper provides an efficient and reliable networked optimal control design of a two-wheeled inverted pendulum system to achieve wide-range system stabilization. Linear quadratic regulator (LQR) theory and its modified version with exponential term are initially cast into the LMI framework to guarantee stabilization as a self-balancing robot. Next, an integral action is inserted into the LQR performance index to decrease the error substantially. Finally, the event-based policy is applied alongside LQR to decrease communication congestion over the network. An observer-based control is used; once the states are estimated, an optimal control law is generated to regulate the torque of the motors. Simulation results demonstrate that the system performance still optimal while the event-based regime is used.

I. INTRODUCTION

Two-wheeled robots have become a popular problem in control literature and mechanics field, like inverted pendulum. Because its mobility that is superior to three and four wheeled automobiles, the importance of balancing TWIP robots in autonomous robot research is on the rise [1, 2]. The TWIP vehicles can be categorized into non-coaxial and coaxial [3]. It is called non-coaxial vehicle when the two wheels are not placed on the same axis. Generally speaking, this kind of vehicles are very common like scooters as well as bicycles. To control this kind of vehicles, the controller should coordinate the horizontal balancing. On the contrary, the coaxial vehicles comprises wheels placed on the same axis, which has become a popular transportation vehicle everywhere in recent years.

The research about the stability of the TWIP robot is very active since the past two decades. The TWIP is an inherently unstable nonlinear system [4], which makes it an appropriate system as an experimental device for research in order to test the advanced control methodologies [5]. A linear feedback control is used to stabilize the system of inverted pendulum when the uncertainty is not considered [6, 7]. As well as, feedback linearization controllers perform well when the angle of the pendulum is small [8]. In order to include the uncertainty, one can apply adaptive control [9, 10], observer method for disturbance rejection [11], or sliding mode control methodology [12].

The Two-Wheeled Cehicle (CTWEV) is belonging to the coaxial type of vehicles. Figure 1 illustrates the wheels configuration of the vehicle. Early results on balancing two-coaxial-wheeled robot was reported in [13]. The research presented in [14], motivated a lot of today's research as can be shown in the subsequent studies [15, 16, 17, 18]. The CTWEV is used in many applications, such as transportation, entertainment, security, and military affairs. The CTWEV is inherently unstable dynamical system that looks like an inverted mobile pendulum. However, its dynamics are much more sophisticated than its conventional counterparts of the inverted pendulum are.

The TWIP is of a kind of mobile robots, which its motion and balancing upward is implemented using many techniques such as linear feedback, exact feedback linearization, ro-

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bust control, and Lyapunov control [8, 19, 20, 21, 22]. Researches [23] focus on the automobile vehicle system and demonstrate the viability of their construction. Many of the strategies used to stabilize the TWEV obtained from the full information control setup.

In recent studies, gyroscopes are very important because the measurements of angular velocity and acceleration is crucial to perform control methods using full information analysis. Despite of the fact that full information approach stabilizes the internal modes of the plant using constant feedback control, it requires a bunch of sensors, which increases the cost tremendously. For instance, the system of Segway requires five gyroscopes for velocity and two inclinometers for position. Since the two-wheeled vehicle is a multi-input and multi-output system, the construction of full state control inevitably needs an observer base controller if the sensors are neglected.

It should be noted that the acceleration measurements is not necessary in the observer-based controllers. Because the expensive cost of inverted two wheeled sensors, the design of these control systems are not practical solutions when the cost is considered. These methods are always reliable for control, but the cost issues could affect the design of the controller and increase the complexity. Over the last decade technological advancements have led to the development of low cost sensors/actuators and communication devices that can be used to control remote plants, related approaches can be found in [24, 25, 26]. In recent years, event-triggering policy has established itself as a major advance technology [27, 28, 29, 30, 31, 32]. It has been widely implemented in many trends due to its advantages in decreasing the communication congestion between the controller and the plant. The development of intelligent autonomous vehicle and remote-controlled wheeled robots are noteworthy. In this note, manipulation and control of the CTWEV using event triggering mechanism will also be examined.

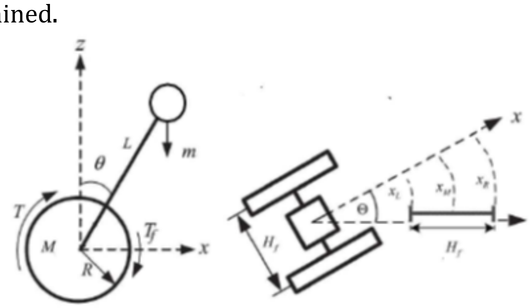

Fig. 1. The top and side view of vehicle

II. ROBOT MODELING

Figure 1 illustrates the side and top view of the TWIP robot. The parameters θ and Θ are the pitch angle, and the path turning angle, restrictively, whereas M, M_w, m, and R are the robot body mass, the wheel mass, payload mass, and wheel radius, respectively. The mathematical model if the TWIP robot is obtained using Lagrangian method. The Lagrangian function Lhas the form:

$$\mathbf{L} = \mathbf{T} - \mathbf{U}$$

where T is the kinetic energy and U is the potential energy.

$$\begin{aligned} T &= \frac{1}{2}\left(M + 2M_w\right)\dot{x}_M^2 + \frac{1}{2}m\left(\dot{x}_M + \dot{\theta} l \cos(\theta)\right)^2 \\ &= +\frac{1}{2}m(-\dot{\theta} l \sin(\theta))^2 + \frac{1}{2}\left(J_{m\theta} + J_{p\theta}\right)\dot{\theta}^2 \\ &= +\frac{1}{2}\left(2J_w\right)\left(\frac{\dot{x}_M}{R}\right) + \frac{1}{2}J_\Theta\dot{\Theta}^2 \\ &= +\frac{1}{2}\left[\left(\frac{2J_w}{R^2} + 2M_w\right)\left(\frac{H_f}{2}\right)^2\right]\dot{\Theta}^2 \\ U &= -mg(l - l\cos(\theta)) \end{aligned}$$

The Lagrangian equations are:

$$\cdot\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x} M}\right) - \frac{\partial L}{\partial x_M} = \frac{T_l}{R} + \frac{T_R}{R}$$
$$\cdot\frac{d}{dt}\left(\frac{\partial L}{\partial \theta}\right) - \frac{\partial L}{\partial \theta} = 0$$
$$\cdot\frac{d}{dt}\left(\frac{\partial L}{\partial \Theta}\right) - \frac{\partial L}{\partial \Theta} = (T_L - T_R)\frac{H_I}{R}$$

Where $x_M, J_\ominus, J_{m\theta}, J_{p\theta}, J_w, H_f, 1, T_L, T_R$ are the trajectory of the center cart, whole cart moment or inertia around z-axis, mainframe moment of inertia, pendulum moment of inertia, single wheel moment of inertia, length between the wheels, left motor torque, and right motor torque, respectively. Then, we can obtain the nonlinear model using the Lagrangian equations:

$$\ddot{\theta} = \frac{\frac{ml\cos(\theta)\left(\frac{\tau_l}{\mu} + \frac{T_R}{\mu} + ml\dot{\theta}^2\sin(\theta)\right)}{M + 2M_v + m + \frac{2J_w}{R^2}} - mgl\sin(\theta)}{\frac{m^2 l^2\cos^2(\theta)}{M + 2M_w + m + \frac{2J_J}{R^2}} - (ml^2 + J_{m\theta} + J_{p\theta})}$$

$$\ddot{x}_M = \frac{\frac{T_L}{R} + \frac{T_R}{R} + ml\dot{\theta}^2\sin(\theta) - \frac{m^2 l^2 g\sin(\theta)\cos(\theta)}{ml^2 + J_{mb} + J_{p\theta}}}{M + 2M_w + m + \frac{2J_w}{R^2} - \frac{m^2 + l^2\cos(\theta^2)}{m^2 + J_m\theta + J_{p\theta}}}$$

$$\ddot{\Theta} = \frac{H_f}{J_\Theta + \frac{(J_w + R^2 M_w)H_l^2}{2R^2}}\left(\frac{T_L}{R} - \frac{T_R}{R}\right)$$

The linearized model of the cart system at the equilibrium point is given by:

$$\ddot{\theta} = \frac{-mglM_e}{m^2 l^2 - (ml^2 + J_e)M_e}\theta + \frac{ml(T_L + T_R)}{Rm^2 l^2 - RM_e(ml^2 J_e)}$$
$$\ddot{x}_M = \frac{-\frac{m^2 l^2 g}{ml^2 + J_e}}{M_e - \frac{m^2 l^2}{ml^2 + J_e}}\theta + \frac{(T_L + T_R)}{R\left(M_e - \frac{m^2 l^2}{ml^2 + J_e}\right)}$$
$$\ddot{\Theta} = \frac{2RH_f(T_L - T_R)}{2R^2 J_\Theta + (J_w + R^2 M_w)H_f^2}$$

Where $M_e = M + 2M_w + m + \dfrac{2J_x}{R^2}$ and $J_e = J_{m\theta} + J_{p\theta}$

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The linearized model is used to describe the dynamic behavior at the equilibrium point of the vehicle which can be written in the standard form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{1}$$

Where $A \in \mathbb{R}^{6\times 6}, B \in \mathbb{R}^{6\times 2}$, and $C \in \mathbb{R}^{3\times 6}$. The input of the system is given by the torque values of the right and left motors of the TWEV, i.e., $u = \begin{bmatrix} T_L & T_R \end{bmatrix}^T$. The states of the plant is given by $x = [x_1, x_2, x_3, x_4, x_5, x_6]^T$, where $x_1 = \theta, x_2 = \dot{\theta}, x_3 = x_M, x_4 = \dot{x}_M, x_5 = \Theta$, and $x_6 = \dot{\Theta}$ Then the matrices of the linearized dynamics are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-mglM_c}{m^2l^2-(ml^2+J_e)M_e} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{-m^2l^2g}{M_e-\frac{ml^2+J_c}{ml^2+J_e}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ \frac{ml}{Rm^2l^2-RM_e(ml^2J_e)} & \frac{ml}{Rm^2l^2-RM_e(ml^2J_e)} \\ 0 & 0 \\ \frac{1}{R\left(M_e-\frac{m^2|2}{m^2+J_e}\right)} & \frac{1}{R\left(M_e-\frac{m^2|^2}{m'+J_e}\right)} \\ 0 & 0 \\ \frac{2RH_f}{2R^2J_\Theta+(J_e+R^2M_\nu)H_j^2} & -\frac{2RH_f}{2R^2J_\Theta+(J_\nu+R^2M_\nu)H_j^2} \end{bmatrix}$$

In this paper, we apply various control methods to stabilize the TWEV. An observer-based control is used to control the plant, which means an observer must be designed. Before applying the control methods, we have to study some characteristics such as controllability and observability. The parameters of the system are taken from [13], which are given by $J_{m\theta} = 0.153(\text{Kgm2}), J_{p\theta} = 0.125(\text{Kgm2})$ $M_w = 5.44(\text{Kg}), M = 15.747(\text{Kg}), H_f = 0.44(\text{m}), m = 4(\text{Kg}), I = 0.53(m), J_w = 0.013(Kgm2), J_\Theta = 0.576(Kgm2)$, and $R = 0.1(\text{m})$ Then the matrices of the state space model 1 are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 16.85 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -1.3418 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ -0.6458 & -0.6458 \\ 0 & 0 \\ 0.427 & 0.427 \\ 0 & 0 \\ 3.5815 & -3.5815 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The linearized model is unstable and non-minimum phase system. Moreover, it is completely controllable and observable. This allows us to design the controllers using full state feedback in terms of observer-based control. The discrete time model of the linearized two-wheeled system has the form

$$\begin{aligned} x_{k+1} &= A_d x_k + B u_k \\ y_k &= C x_k \end{aligned} \tag{2}$$

Where $\boldsymbol{A}_d = e^{\boldsymbol{\Lambda T}}, \boldsymbol{B}_d = \boldsymbol{J_0^T} e^{\boldsymbol{\Lambda}\tau} \boldsymbol{d}\tau \boldsymbol{B}$, and $\boldsymbol{C_d} = \boldsymbol{C}$

The sampling time $T_s = 0.001$. Then the matrices in the discrete form is expressed by:

$$A_d = \begin{bmatrix} 1.0008 & 0.01 & 0 & 0 & 0 & 0 \\ 0.1685 & 1.0008 & 0 & 0 & 0 & 0 \\ -0.671 & -0.0022 & 1 & 0.01 & 0 & 0 \\ -134.21 & -0.6710 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.01 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$B_d = \begin{bmatrix} 0 & 0 \\ -0.0065 & -0.0065 \\ 0 & 0 \\ 0.0057 & 0.0057 \\ 0.0002 & -0.0002 \\ 0.0358 & -0.0358 \end{bmatrix} \tag{3}$$

III. CONTROL DESIGN

With reference to Figure 2, we develop in the sequel an event-based networked control scheme based on LQR theory.

A. Discrete-Time LQR Control of Two-Wheeled System

In this section, we stabilize the process using LQR. The performance index of the linear quadratic control is expressed as.

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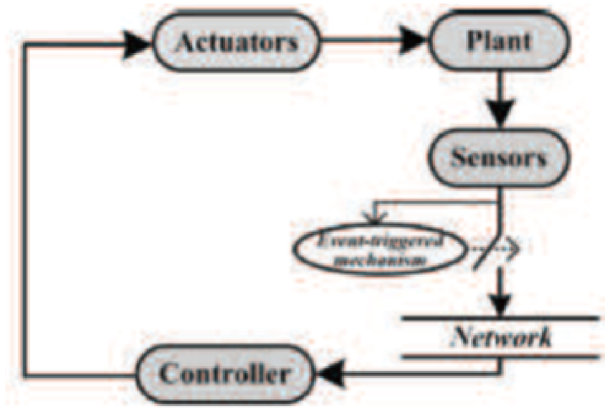

Fig. 2. Event based network control system

$$J=\sum_{k=0}^{\infty}\left(x_k^T Q x_k+u_k^T R u_k\right) \quad (4)$$

where $Q \geq 0$ and $R > 0$ We need to stabilize the process from any initial state x (0) such that the cost is minimum. Enormous research has been done on the LQR problem. Fortunately, the optimal control law generated by static feedback gain with this form:

$$u_k = Kx_k$$

In what follows, we use an LMI-formulation to the LQR problem of dynamics (2) while minimizing the performance index (4). Our methodology is to determine a linear optimal static-feedback control $u_k = Kx_k$ that achieves this goal.

1) *Assumption 1* : Assume that there exist a positive definite function $V(x_k) = x_k P x_k$ such that there exists $\gamma^* \in \mathbb{R}^+$ that satisfies $x_0^T P x_0 \leq \gamma^*$.
2) *Remark 1* : The upper bound γ^* used to minimize the performance cost, i.e., we proceed to minimize γ^* instead of directly minimizing $x_0^T P_{x_0}$.

3) *Theorem 1* : Given positive definite matrices $R > 0$ and $Q > 0$. The control law

$$u_k = Kx_k$$

asymptotically stabilizes the discrete time system (2) with a guaranteed cost $J_{oo} \leq \gamma^*$ if there exist matrices $X > 0, Z$ such that the following LMI optimization problem is feasible,

$$\min \gamma \begin{bmatrix} -\mathcal{X} & (A\mathcal{X}+BZ)^T & \mathcal{Z}^T & \mathcal{X} \\ \bullet & -\mathcal{X} & 0 & 0 \\ \bullet & \bullet & -R^{-1} & 0 \\ \bullet & \bullet & \bullet & -Q^{-1} \end{bmatrix} \leq 0 \quad (5)$$

$$\begin{bmatrix} \gamma^* & x_0^T \\ x_0 & \mathcal{X} \end{bmatrix} \geq 0 \quad (6)$$

Moreover, the controller gain is given by $K = ZX^{-1}$.

4) *Proof 1* : Using Schur complement, it is straightforward to show that inequality (6) implies that

$$V(x_0) = x_0^T P x_0 \leq \gamma^* \quad (7)$$

Pre-and post-multiply (22) by diag$\{P, I, I, I\}$, and using $\mathcal{X} = P^{-1}, \mathcal{Z} = \mathcal{K}P^{-}1$, it follows that (22) is equivalent to

$$\begin{bmatrix} -P & (A+BK)^T & K^T & I \\ \bullet & -P^{-1} & 0 & 0 \\ \bullet & \bullet & -R^{-1} & 0 \\ \bullet & \bullet & \bullet & -Q^{-1} \end{bmatrix} \leq 0 \quad (8)$$

Additionally, using Sure complement, inequality (8) can be expressed as

$$-P + \Theta^T \Pi^{-1} \Theta \leq 0 \quad (9)$$

Where

$$\Theta = \begin{bmatrix} (A+BK)^T & K^T & I \end{bmatrix} \quad (10)$$

$$\Pi = \begin{bmatrix} -P^{-1} & 0 & 0 \\ \bullet & -R^{-1} & 0 \\ \bullet & \bullet & -Q^{-1} \end{bmatrix} \quad (11)$$

Multiplying both sides of equation (9)by x_k gives

$$x_k^T\left[(A+BK)^T P(A+BK) - P\right]x_k \leq -x_k^T\left[Q + K^T R K\right]x_k \quad (12)$$

Alongside the dynamics of the system (2), substituting $u_k = kx_k$ and the Lyapunov function in Assumption (1) gives

$$\Delta V(x) = V(x_{k+1}) - V(x_k) \leq -\left(x_k^T Q x_k + u_k^T R u_k\right)$$
(13)

the right hand side of (13) is negative definite since $Q > 0$ which implies that the the discrete time system is asymptotically stable, i.e., $x_k \rightarrow$ o. Take the infinite sum to both sides of Equations (13) gives

$$\begin{aligned} &\lim_{N\to\infty}\sum_{0}^{N}\left[V(x_{k+1}) - V(x_k)\right] \\ =&\lim_{N\to\infty}\left[V(x_{N+1}) - V(x_0)\right] \\ =&0 - V(x_0) \\ \leq& -\lim_{N\to\infty}\sum_{0}^{N}\left(x_k^T Q x_k + u_k^T R u_k\right) \\ =& -J_{\infty} \end{aligned} \quad (14)$$

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Then using Equations (7) and (14)

$$J_\infty \leq V(x_0) \leq \gamma^\star$$

This completes the proof.

B. Modified LQR with Integral Action

The performance index can be modified by including the integral action of the output. Define the integral action on the outputs as a new variable:

$$z_k = \sum_{n}^{k-1} y_i \tag{15}$$

The proportional-integral control law becomes.

$$u_k = \begin{bmatrix} K_p & K_I \end{bmatrix} \begin{bmatrix} x_k \\ z_k \end{bmatrix} \tag{16}$$

Then the variable z_k can be added in its quadratic form to the performance index. For positive definite matrices $R > 0, S > 0$, and $Q > 0$, the modified performance index is given by

$$J = \sum_{k=0}^{\infty} \left(x_k^T Q x_k + u_k^T R u_k + z_k^T S z_k \right) \tag{17}$$

To solve this problem, we use the augmented vector

$$\xi_k = \begin{bmatrix} x_k & z_k \end{bmatrix}$$

Which creates a new state space model of the form:

$$\xi_{k+1} = \overline{\boldsymbol{A}} \xi_k + \overline{\boldsymbol{B}} u_k \tag{18}$$

$$y_k = \bar{C} \varepsilon_k \tag{19}$$

Where

$$\bar{A} = \begin{bmatrix} A & 0 \\ C & I \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad \bar{C} = \begin{bmatrix} C & 0 \end{bmatrix}$$

Then the performance index (17) can be written as

$$J = \sum_{k=0}^{\infty} \left(\xi_k^T \bar{Q} \xi_k + u_k^T R u_k \right) \tag{20}$$

Where

$$\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & S \end{bmatrix} \tag{21}$$

Following the same argument in (1) yields the following corollary, which is used to obtain the proportional integral action.

1) Corollary 1 : Given positive definite matrices $R > 0$ and $Q > 0$. The control law

$$u_k = K_p x_k + K_I z_k$$

asymptotically stabilizes the discrete time system (2) with a guaranteed cost

$$J_\infty \leq \gamma^*$$

if there exist matrices $X > 0$, Z, such that the following LMI optimization problem is feasible,

$$\min \gamma \begin{bmatrix} -\mathcal{X} & (\bar{A}\mathcal{X} + \bar{B}\mathcal{Z})^T & \mathcal{Z}^T & \mathcal{X} \\ \bullet & -\mathcal{X} & 0 & 0 \\ \bullet & \bullet & -R^{-1} & 0 \\ \bullet & \bullet & \bullet & -Q^{-1} \end{bmatrix} \leq 0 \tag{22}$$

$$\begin{bmatrix} \gamma^* & \xi_0^T \\ \xi_0 & \mathcal{X} \end{bmatrix} \geq 0 \tag{23}$$

The controller gain is given by $\begin{bmatrix} K_p & K_I \end{bmatrix} = \mathcal{Z}\mathcal{X}^{-1}$

IV. EVENT-BASED CONTROL

Consider the discrete time system of the form

$$x(k+1) = Ax(k) + Bu(k) \tag{24}$$

The system is assumed stabilizable by constant state feedback

$$u_k = K x_k$$

However, the input of the plant with event trigger strategy is

$$u(k) = K\hat{x}(k)$$

Where $\hat{x}(k)$ is the last transmitted signal. Let the error between the last transmitted signal and the actual one be

$$e(k) = x(k) - \hat{x}(k)$$

Then the received control law becomes

$$u(k) = Kx(k) + Ke(k)$$

The system described in Equation (24) becomes

$$x(k+1) = (A + BK)x(k) + BKe(k) \tag{25}$$

The system has the Input to State Stable (ISS) property if there exist a positive definite function and class K_∞ functions

$$\alpha_1(\|x\|), \alpha_2(\|x\|), \alpha(\|x\|) \text{ and } \gamma(\|e\|)$$

Such that the following conditions are satisfied

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|) \tag{26}$$

$$V(x(k+1)) - V(x(k) \leq -\alpha(\|x\|) + \gamma(\|e\|) \tag{27}$$

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2) Theorem 2: [18] The system considered in (25) is asymptotically stable using the event-triggered rule:

$$\gamma(\|e\|) \leq \sigma\alpha(\|x\|)$$
$$0 < \sigma \leq 1$$

To use Theorem 2 in our case, we use the discrete Lyapunov function $V(x) = x^\top(k)Px(k)$, where $P = X^{-1}$, and using simple calculation of the difference of Lyapunov function, Equation (27) becomes

$$\begin{aligned} x^T(k)[(A+ BK)^T P(A+BK) - P]\, x(k) \\ + e^{(k)} \left[K^T B^T PBK\right] e(k) \\ + 2x^T(k)[(A+BK)PBK]e(k) \\ \leq -\alpha(\|x\|) + \gamma(\|e\|) \end{aligned} \quad (28)$$

Then, we can choose the following class K_{00} functions:

$$\alpha(\|x\|) = x^T(k)\mathcal{S}x(k)$$
$$\gamma(\|e\|) = e(k)^T \mathcal{W} e(k)^T$$

Where S and W are positive definite matrices. Plugging $\alpha(\|x\|)$ and $\gamma(\|e\|)$ into Equation (28) gives

$$\begin{bmatrix} x(k)^T & e(k)^T \end{bmatrix} \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \cdot & \Omega_{22} \end{bmatrix} \begin{bmatrix} x(k)^T \\ e(k)^T \end{bmatrix} \leq 0 \quad (29)$$

Where

$$\Omega_{11} = (A+BK)^T P(A+BK) - P + \sigma S$$
$$\Omega_{12} = (A+BK)^T PBK$$
$$\Omega_{22} = K^T B^T PBK - \mathcal{W}$$

The formula of triggering can be written as

$$\|e\| \leq \mu\|x\| \quad (30)$$

Where $\mu = \sigma\lambda_{\max}(S)/\lambda_{\min}(W)$ Equation (29) comprises a linear matrix inequality since the S and W are the only variables.

V. SIMULATION RESULTS

In this section, we demonstrate the simulation results obtained by applying Theorem 1 and its corresponding Corollary 1. Figure 3 shows the response of the output of the two-wheeled vehicle which stabilized according to the LQR method without the integration term.

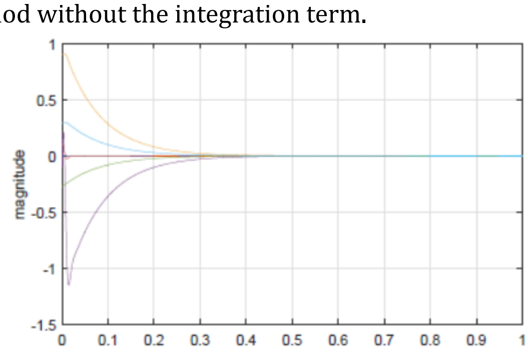

Fig. 3. The output response of the LQR control

On the other hand, Figure 4 demonstrate the response of the model when the integral action is considered. We see that the integral action forces the response to converge to the equilibrium point in a short time.

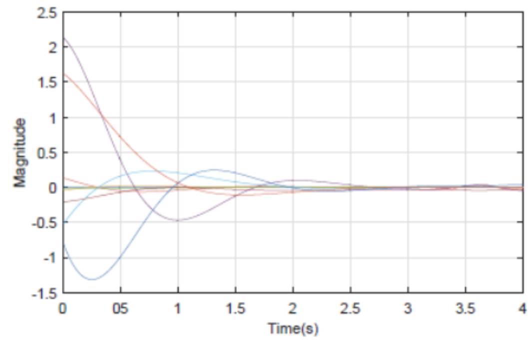

Fig. 4. The integral action response

The event triggering rule in (30) is obtained as illustrate in the note. Using event-triggering mechanism, the response of the two-wheeled vehicle is still a stable system with some degradation of output performance as can be seen in Figure 5.

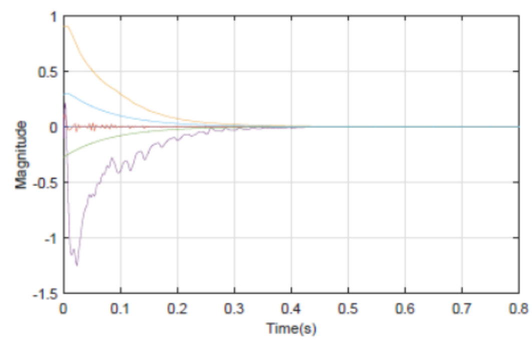

Fig. 5. Event based control

VI. CONCLUSION

This paper has examined optimal control schemes based on LQR applied for a two wheeled-inverted pendulum. The plant in this note is inherently unstable and multi-input multi-output system. Standard LQR and its modified version using an integral action are utilized to stabilize the plant. Moreover, event-triggering rule is used to decrease the total number of transmissions when the vehicle is to be controlled remotely. Finally, simulation results are used to evaluate the effectiveness and feasibility of our strategies.

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